A HOSPITAL ADMISSION PROBLEM

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Formulation

Setting aside beds required for urgent cases and beds for those already admitted or obligated for assignment, there is available at the start of any day t, a_t beds where $t = 0,1,2,\ldots,T$. The day T represents a planning horizon of say $T = 30$ days in the future.

We shall characterize an admission request by

(a) the earliest date the patient could be admitted;

(b) the latest date to which he can be postponed;

(c) the length of his stay.

Two patients having the same values for all three of these aspects (a, b, c) will be said to belong to the same admission type j. Thus the same length of stay but a different interval of possible admission dates constitutes a different type patient. Let d_j be the number of patients of each type j (estimated) who will call in for admission.

Let us suppose that d_j is known and that the only question we are asking is: how many of the d_j should be assigned to starting date s ? Call this amount to be determined x_{js}. It is clear that

(1)

$$x_{j\infty} + \sum_{s=f_j}^{g_j} x_{js} = d_j$$

where $x_{j\infty}$ are the number turned away and $f_j \leq s \leq g_j$ represents the range of possible starting dates of j .

It is natural to want to assign differing costs c_{js} for different starting dates. Presumably $c_{j\infty}$ would be very high to reflect the undesireability of turning away a patient and also that c_{jt} will monotonically increase in both directions away from some ideal starting date within its range.

Let $\delta_t(j, s)$ be defined as a coefficient for x_{js} whose value is either 1 or 0 depending on whether or not a patient of type j who starts at time s will be using a bed at time t . For example if he is a patient whose length of stay is 3 days, then $\delta_t(j, s) = 1$ if $s \leq t < s + 3$ otherwise $\delta_t(j, s) = 0$.

Since available bed capacity cannot be exceeded, we have for $t = 0,1,\ldots,T$,

(2)

$$\sum_{j,s} x_{js}\,\delta_t(j, s) \leq a_t .$$

It is also possible to introduce above slack variables (excess and deficit) and assign appropriate costs (loss revenues, inconvenience in case of emergency.)

Special Structure

The entire model when cast in this form is a linear program with a very special structure which we now proceed to study. In detached coefficients, it will be noted that the model is like a classical transportation problem in that there are a set of equations (1) representing demands (d_j) and equations (2) representing availabilities (a_t) . The essential difference is that each unit of demand, if admitted, places a lien against availabilities in successive days. Hence the basic model takes the form illustrated below:

(I)	1	1	1	1													$= d_1$
(II)					1	1	1										$= d_2$
(III)								1	1	1	1	1					$= d_3$
(IV)	1				1			1					1				$= a_1$
(V)	1	1			1	1		1	1					1			$= a_2$
(VI)		1	1		1	1		1	1	1					1		$= a_3$
(VII)			1			1			1	1	1					1	$= a_4$
	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{41}	c_{42}	c_{43}	c_{44}	
	type 1				type 2			type 3					slack avail.				

Thus type $j = 1$ can start on day 1,2, or 3 and has a two day stay; type $j = 3$ can start on day 1,2,3, or 4 and has a three day stay.

This can always be simplified by successive subtractions of the second set of equations. The right-hand column indicates the operations used.

$$
\begin{array}{l ccc|cc|cccc|cccc|l}
IV' & 1 & & & 1 & & 1 & & & & 1 & & & & = a_1 \\
V' & & 1 & & & 1 & & 1 & & & -1 & 1 & & & = a_2 - a_1 \\
VI' & -1 & & 1 & & & & & 1 & & & -1 & 1 & & = a_3 - a_2 \\
VII' & & -1 & & -1 & & -1 & & & 1 & & & -1 & 1 & = a_4 - a_3
\end{array}
$$

It is clear that equations IV, V, VI, VII can be replaced by those with primes. The latter is characterized by the well-known network formulation.

Solution

It is very well worth studying this case of structures for its own sake. It would be no surprise if one should come up with an extremely simple solution technique. I shall now discuss a less special approach to this problem based on generalized upper bounding in linear programming. [1], [2].

Note that the second set of equations are relatively few in number, namely T where $T = 30$. The upper set of demand equations is characterized, however, by equations whose variables are mutually exclusive of each other. The number of these could be quite large. For instance, five different spreads of admittance with 10 different number of days of stay would give rise to 50 different types per day or a total of 1,500 equations of the d_j type in a thirty day study.

Essentially, it is proposed that the entire system be treated like a 30 + 30 + 1 = 2T + 1 linear program and solved by the "generalized" upper-bound algorithm, [1], [2].

Two comments

First, fractional values for x_{js} can occur. These will be rare and a satisfactory scheme of rounding should not be difficult to devise. Second, we have formulated the problem assuming the values of d_j are known in advance. Actually we do not know this. The above theory can be extended, however, to cover the case where the distribution of d_j only is known. Only minor changes in formulation and computational effort would be required, [3].

Interpretation of the Solution

The scheduling clerk could be given a prepared list with an ideal starting date <u>page</u>, a range <u>section</u> and a length-of-stay <u>line</u>. The entry gives the number that may be assigned on various alternative starting dates until none are available. For her purpose it may be simpler to consolidate all the range sections together on a page. In fact, from the point of view of feasibility, one may wish to consider as final output, a tray containing a two way array of boxes containing cards. Each card has on it a starting date and an estimated length of stay.

LENGTH OF STAY

Starting date	1 day	2 days					10 days
6/1	//	/////					
6/2		///				///	
			///				
				//			
6/30	//						///

TRAY

← 3 cards in box

Operationally all the clerk does is to find a box with cards which has the correct length-of-stay on it and a date that is as close to ideal starting date as possible. She pulls the card and writes the patient-doctor's name on it. The tray probably will need to be updated once a week (but this is only a guess).

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[3] DANTZIG, G.B. Linear Programming and Extension, Chapter 25, Programming Under Uncertainty, Princeton University Press, 1963.